

Decays $D_{(s)}^+ \rightarrow \pi(K)^+ \ell^+ \ell^-$ and $D^0 \rightarrow \ell^+ \ell^-$ in the MSSM with and without R-parity

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Abstract

We study the rare decays $D^+ \rightarrow \pi^+ \ell^+ \ell^-$, $D_s^+ \rightarrow K^+ \ell^+ \ell^-$ and $D^0 \rightarrow \ell^+ \ell^-$ ($\ell = e, \mu$) in the minimal supersymmetric standard model with and without R-parity. Using the strong constraints on relevant supersymmetric parameters from $D^0 - \bar{D}^0$ mixing and $K^+ \rightarrow \pi^+ \nu \bar{\nu}$ decay, we examine constrained supersymmetry contributions to relevant branching ratios, direct CP violations and ratios of $D_{(s)}^+ \rightarrow \pi(K)^+ \mu^+ \mu^-$ and $D_{(s)}^+ \rightarrow \pi(K)^+ e^+ e^-$ decay rates. We find that both R-parity conserving LR as well as RL mass insertions and R-parity violating squark exchange couplings have huge effects on the direct CP violations of $D_{(s)}^+ \rightarrow \pi(K)^+ \ell^+ \ell^-$, moreover, the constrained LR and RL mass insertions still have obvious effects on the ratios of $D_{(s)}^+ \rightarrow \pi(K)^+ \mu^+ \mu^-$ and $D_{(s)}^+ \rightarrow \pi(K)^+ e^+ e^-$ decay rates. The direct CP asymmetries and the ratios of $D_{(s)}^+ \rightarrow \pi(K)^+ \mu^+ \mu^-$ and $D_{(s)}^+ \rightarrow \pi(K)^+ e^+ e^-$ decay rates are very sensitive to both moduli and phases of relevant supersymmetric parameters. In addition, the differential direct CP asymmetries of $D_{(s)}^+ \rightarrow \pi(K)^+ \ell^+ \ell^-$ are studied in detail.

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1 Introduction

In the standard model (SM), rare decays $D^+ \rightarrow \pi^+\ell^+\ell^-$, $D_s^+ \rightarrow K^+\ell^+\ell^-$ and $D^0 \rightarrow \ell^+\ell^-$ are induced by $c \rightarrow u\ell^+\ell^-$ flavour changing neutral current (FCNC). Unlike K and B systems, the short distance (SD) contributions to charmed-meson FCNC processes are highly suppressed due to the stronger GIM mechanism and weaker quark mass enhancements in the loops. Therefore, these rare decays definitely could be good candidates to probe new physics (NP) effects. Since the values in the SM are hardly reachable at present D factories, if any exotic event is found, it must be a strong evidence for NP. Many extensions of the SM, such as supersymmetric models with R-parity violation or models involving a fourth quark generation, introduce additional diagrams that a priori need not be suppressed in the same manner as the SM contributions [1–7].

In the following, we will concentrate on $D^+ \rightarrow \pi^+\ell^+\ell^-$, $D_s^+ \rightarrow K^+\ell^+\ell^-$ and $D^0 \rightarrow \ell^+\ell^-$ decays in the minimal supersymmetric standard model (MSSM) with and without R-parity. Using the strong constraints on relevant supersymmetric parameters from $D^0 - \bar{D}^0$ mixing and $K^+ \rightarrow \pi^+\nu\bar{\nu}$ decay, we will examine the susceptibilities of relevant dileptonic invariant mass spectra, branching ratios, differential direct CP violation, direct CP violation and ratios of $D_{(s)}^+ \rightarrow \pi(K)^+\mu^+\mu^-$ and $D_{(s)}^+ \rightarrow \pi(K)^+e^+e^-$ decay rates to the constrained supersymmetric coupling parameters. Our results indicate that the R-parity violating (RPV) contributions and R-parity conserving (RPC) mass insertion (MI) contributions could greatly change the direct CP violations of $D_{(s)}^+ \rightarrow \pi(K)^+\ell^+\ell^-$, furthermore, LR and RL MI contributions also could obviously affect ratios of $D_{(s)}^+ \rightarrow \pi(K)^+\mu^+\mu^-$ and $D_{(s)}^+ \rightarrow \pi(K)^+e^+e^-$ decay rates. These two kinds of quantities are very sensitive to both moduli and phases of relevant supersymmetric parameters. We find that the LD SM effects on all branching ratios except $\mathcal{B}(D^0 \rightarrow \mu^+\mu^-)$ exceed the largest RPV contributions, but for the direct CP violations of $D_{(s)}^+ \rightarrow \pi(K)^+\ell^+\ell^-$, the RPV and RPC contributions could be totally over the SM LD ones.

The paper is organized as follows. In Sec. 2, we derive the expressions for $D_{(s)}^+ \rightarrow \pi(K)^+\ell^+\ell^-$ and $D^0 \rightarrow \ell^+\ell^-$ processes in the MSSM with and without R-parity. In Sec. 3, our numerical analysis are presented. We use the constrained parameter spaces from $D^0 - \bar{D}^0$ mixing and $K^+ \rightarrow \pi^+\nu\bar{\nu}$ decay to present the RPV and RPC effects on the observables of the relevant D decays. Sec. 4 is devoted to our summary.

2 The theoretical framework for $D_{(s)}^+ \rightarrow \pi(K)^+ \ell^+ \ell^-$ and $D^0 \rightarrow \ell^+ \ell^-$ decays

2.1 The semileptonic decays $D \rightarrow P \ell^+ \ell^-$

The effective Hamiltonian for the $c \rightarrow u \ell^+ \ell^-$ transition can be written as [7–9]

$$\mathcal{H}_{eff}^{SM} = -\frac{G_F}{\sqrt{2}} V_{cb}^* V_{ub} \sum_{i=7,9,10} (C_i \mathcal{O}_i + C'_i \mathcal{O}'_i), \quad (1)$$

where four-fermion operators are

$$\begin{aligned} \mathcal{O}_7 &= \frac{e}{8\pi^2} m_c F_{\mu\nu} \bar{u} \sigma^{\mu\nu} (1 + \gamma_5) c, \\ \mathcal{O}_9 &= \frac{e^2}{16\pi^2} \bar{u}_L \gamma_\mu c_L \bar{\ell} \gamma^\mu \ell, \\ \mathcal{O}_{10} &= \frac{e^2}{16\pi^2} \bar{u}_L \gamma_\mu c_L \bar{\ell} \gamma^\mu \gamma_5 \ell, \\ \mathcal{O}'_7 &= \frac{e}{8\pi^2} m_c F_{\mu\nu} \bar{u} \sigma^{\mu\nu} (1 - \gamma_5) c, \\ \mathcal{O}'_9 &= \frac{e^2}{16\pi^2} \bar{u}_R \gamma_\mu c_R \bar{\ell} \gamma^\mu \ell, \\ \mathcal{O}'_{10} &= \frac{e^2}{16\pi^2} \bar{u}_R \gamma_\mu c_R \bar{\ell} \gamma^\mu \gamma_5 \ell, \end{aligned} \quad (2)$$

with $F_{\mu\nu}$ is the electromagnetic field strength, while $q_{L(R)} = \frac{1}{2}(1 - (+)\gamma_5)q$ is the left(right)-handed quark field. The RPC and RPV MSSM effects are including in the Wilson coefficients corresponding to the $D \rightarrow P \ell^+ \ell^-$ decay via

$$\begin{aligned} C_i &= C_i^{SM} + C_i^{\mathcal{R}_p} + C_i^{\mathcal{G}}, \\ C'_i &= C_i^{\mathcal{G}}, \end{aligned} \quad (3)$$

and the Wilson coefficients are taken at the scale $\mu = m_c$.

In the SM, the expressions of the corresponding Wilson coefficients C_i^{SM} can be found in Refs. [5, 10, 11]. In the MSSM without R-parity, the $c \rightarrow u \ell^+ \ell^-$ process is mediated by the tree-level exchange of down squarks [6, 7]. The relevant Wilson coefficients $C_i^{\mathcal{R}_p}$ are

$$C_9^{\mathcal{R}_p} = -C_{10}^{\mathcal{R}_p} = \frac{\sqrt{2}}{G_F} \frac{4\pi}{\alpha_e} \frac{1}{V_{cb}^* V_{ub}} \sum_k \frac{\tilde{\lambda}'_{i2k} \tilde{\lambda}_{j1k}^*}{4m_{\tilde{d}_{kL}}^2}. \quad (4)$$

where $\tilde{\lambda}'_{irk} = V_{rn}^* \lambda'_{ink}$ [12], V_{rn}^* is the SM CKM matrix element, and λ'_{ijk} is RPV coupling constant. Noted that (s)down-down-(s)neutrino vertices have the weak eigenbasis couplings λ' ,

while charged (s)lepton-(s)down-(s)up vertices have the up quark mass eigenbasis couplings $\tilde{\lambda}'$. Very often in the literature (see e.g. [13–17]), one neglects the difference between λ' and $\tilde{\lambda}'$, based on the fact that diagonal elements of the CKM matrix dominate over nondiagonal ones.

In the MSSM with R-parity, the gluino-squark exchange coupling give dominant contributions to these decays. Within the context of the MI approximation [18, 19], allowing for only one insertion, the relevant Wilson coefficients from the gluino-squark exchange couplings are [5]

$$\begin{aligned} C_7^{\tilde{g}} &= -\frac{8}{9} \frac{\sqrt{2}}{G_F m_{\tilde{q}}^2} \pi \alpha_s \left\{ (\delta_{12}^u)_{LL} \frac{P_{132}(u)}{4} + (\delta_{12}^u)_{LR} P_{122}(u) \frac{m_{\tilde{g}}}{m_c} \right\}, \\ C_9^{\tilde{g}} &= -\frac{8}{27} \frac{\sqrt{2}}{G_F m_{\tilde{q}}^2} \pi \alpha_s (\delta_{12}^u)_{LL} P_{042}(u), \end{aligned} \quad (5)$$

where $u = m_{\tilde{g}}^2/m_{\tilde{q}}^2$ and the functions $P_{ijk}(u)$ are defined as

$$P_{ijk}(u) \equiv \int_0^1 dx \frac{x^i (1-x)^j}{(1-x+ux)^k}. \quad (6)$$

$C_7^{\prime\tilde{g}}$ and $C_9^{\prime\tilde{g}}$ are determined by the expressions of $C_7^{\tilde{g}}$ and $C_9^{\tilde{g}}$ with the replacement $L \leftrightarrow R$, respectively.

The decay amplitudes including the SD contribution for $D \rightarrow P \ell^+ \ell^-$ decays can be written as follows

$$\begin{aligned} \mathcal{A}^{SD}(D(p) \rightarrow P(p-q) \ell^+(p_+) \ell^-(p_-)) &= -i \frac{\alpha_e G_F}{4\pi\sqrt{2}} V_{cb}^* V_{ub} f_+(q^2) \left\{ (C_{10} + C'_{10}) \bar{u}(p_-) \not{p} \gamma_5 v(p_+) \right. \\ &\quad \left. + \left[(C_7 + C'_7) \frac{8m_c}{m_D} + (C_9 + C'_9) \right] \bar{u}(p_-) \not{p} v(p_+) \right\}, \quad (7) \end{aligned}$$

and the form factor $f_+(q^2)$ is defined by [20]

$$\begin{aligned} \langle P(p-q) | \bar{u} \gamma^\mu (1 \pm \gamma_5) c | D(p) \rangle &= (2p-q)^\mu f_+(q^2) + q^\mu f_-(q^2), \\ \langle P(p-q) | \bar{u} \sigma^{\mu\nu} (1 \pm \gamma_5) c | D(p) \rangle &= i s(q^2) [(2p-q)^\mu q^\nu - q^\mu (2p-q)^\nu \pm i \epsilon^{\mu\nu\alpha\beta} (2p-q)_\alpha q_\beta], \quad (8) \end{aligned}$$

with the approximation $s(q^2) = f_+(q^2)/m_D$.

Using the formulae presented above, we give formulas for the dilepton invariant mass spectra

$$\begin{aligned} \frac{d\mathcal{B}^{SD}}{ds} &= \frac{\tau_D \alpha_e^2 G_F^2 |V_{cb}^* V_{ub}|^2 f_+^2(s)}{2^{14} \pi^5 m_D^3} \left\{ |C_{10} + C'_{10}|^2 + \left| (C_7 + C'_7) \frac{8m_c}{m_D} + (C_9 + C'_9) \right|^2 \right\} \\ &\quad \left\{ [(m_D^2 - m_P^2 + s)^2 - 4m_D^2 s] u(s) - \frac{1}{3} u^3(s) \right\}, \quad (9) \end{aligned}$$

with $s \equiv q^2$.

For the long distance (LD) contributions to the $D_{(s)}^+ \rightarrow \pi(K)^+ \ell^+ \ell^-$ decays, we will follow Refs. [7, 21], the long distance contributions can be written by the replacements $C_9 \rightarrow C_9 + C^{LD}$.

$$C^{LD} = \frac{i\sqrt{2}4\pi}{G_F \alpha_e} \frac{1}{V_{cb}^* V_{ub}} \left(\frac{a_\rho}{s - m_\rho^2 + im_\rho \Gamma_\rho} - \frac{1}{3} \frac{a_\rho}{s - m_\omega^2 + im_\omega \Gamma_\omega} \right) - \frac{32\pi^2}{3} \frac{V_{cs}^* V_{us}}{V_{cb}^* V_{ub}} e^{i\delta_\phi} \frac{a_\phi m_\phi \Gamma_\phi}{s - m_\phi^2 + im_\phi \Gamma_\phi}. \quad (10)$$

For $D^+ \rightarrow \pi^+ \ell^+ \ell^-$, $a_\rho = (2.5 \pm 0.2) \times 10^{-9}$ [22] and $a_\phi = 1.23 \pm 0.05$ [21]. For $D_s^+ \rightarrow K^+ \ell^+ \ell^-$, $a_\rho = 6.97 \times 10^{-9}$ [7] and $a_\phi = 0.49 \pm 0.05$ [21]. In addition, $\delta_\phi = 0$ will be used in our analyses.

From Eq. (9), one can obtain the differential direct CP violation [21, 23]

$$a_{CP}(s) = \frac{d\mathcal{B}(D^+ \rightarrow M^+ \ell^+ \ell^-)/ds - d\mathcal{B}(D^- \rightarrow M^- \ell^+ \ell^-)/ds}{d\mathcal{B}(D^+ \rightarrow M^+ \ell^+ \ell^-)/ds + d\mathcal{B}(D^- \rightarrow M^- \ell^+ \ell^-)/ds}, \quad (11)$$

the direct CP violation [21, 23]

$$A_{CP}(D^+ \rightarrow M^+ \ell^+ \ell^-) = \frac{\mathcal{B}(D^+ \rightarrow M^+ \ell^+ \ell^-) - \mathcal{B}(D^- \rightarrow M^- \ell^+ \ell^-)}{\mathcal{B}(D^+ \rightarrow M^+ \ell^+ \ell^-) + \mathcal{B}(D^- \rightarrow M^- \ell^+ \ell^-)}, \quad (12)$$

and the ratio of decay branching ratios of $D \rightarrow P \ell^+ \ell^-$ into dimuons over dielectrons [24]

$$R_M = \frac{\mathcal{B}(D^+ \rightarrow M^+ \mu^+ \mu^-)}{\mathcal{B}(D^+ \rightarrow M^+ e^+ e^-)}. \quad (13)$$

2.2 The leptonic decay $D^0 \rightarrow \ell^+ \ell^-$

The general form for the amplitude of $D^0(p) \rightarrow \ell^+(k_+) \ell^-(k_-)$ is [6]

$$\mathcal{M}(D^0 \rightarrow \ell^+ \ell^-) = \bar{u}(k_-)[A_{D^0 \ell^+ \ell^-} + \gamma_5 B_{D^0 \ell^+ \ell^-}]v(k_+). \quad (14)$$

The vector leptonic operator $\bar{\ell} \gamma_\mu \ell$ does not contribute for on-shell leptons as $p_D^\mu (\bar{\ell} \gamma_\mu \ell) = (p_{\ell^+}^\mu + p_{\ell^-}^\mu)(\bar{\ell} \gamma_\mu \ell) = 0$, i.e., $A_{D^0 \ell^+ \ell^-} = 0$. The associated decay branching ratio is [25]

$$\mathcal{B}(D^0 \rightarrow \ell^+ \ell^-) = \frac{\tau_D m_D}{8\pi} \sqrt{1 - \frac{4m_\ell^2}{m_D^2}} |B_{D^0 \ell^+ \ell^-}|^2. \quad (15)$$

The SM SD contributions in $D^0 \rightarrow \ell^+ \ell^-$ lead to a very suppressed branching ratio. $\mathcal{B}_{SM}^{SD}(D^0 \rightarrow \mu^+ \mu^-)$ is of order $10^{-19} \sim 10^{-18}$ [6, 26–28], while taking into the LD contributions, the branching ratio can reach a level of 10^{-13} [6, 26, 27]. $\mathcal{B}_{SM}^{SD}(D^0 \rightarrow e^+ e^-)$ is of order 10^{-23} [6], which

is much smaller than $\mathcal{B}_{SM}^{SD}(D^0 \rightarrow \mu^+ \mu^-)$. These smallness of the SM signal makes it easier for NP contributions to stand out.

In the RPV MSSM, the coefficient $B_{D^0 \ell^+ \ell^-}^{R_p}$ given in Eq. (14) is [25]

$$B_{D^0 \ell^+ \ell^-}^{R_p} = \sum_k \frac{\tilde{\lambda}'_{i2k} \tilde{\lambda}'_{j1k}^*}{4m_{\tilde{d}_{kL}}^2} i m_\ell f_D. \quad (16)$$

In the RPC MSSM, the squark-gluino contribution to the pure leptonic D decays [6, 8]

$$B_{D^0 \ell^+ \ell^-}^{\tilde{g}} = \frac{i\sqrt{2}G_F}{9\pi} \alpha_s m_\ell f_D [(\delta_{22}^u)_{LR}(\delta_{12}^u)_{RL} P_{032} - (\delta_{22}^u)_{RL}(\delta_{12}^u)_{LR} P_{122}]. \quad (17)$$

The double MI is required to induce a helicity flip in the squark propagator. Noted that the chargino contribution to the Z penguin for $D \rightarrow \ell^+ \ell^-$ also contains a double MI. Due to the double MI, these contributions to $D \rightarrow \ell^+ \ell^-$ is completely negligible, and we will not examine the MI effects in $D \rightarrow \ell^+ \ell^-$ decays.

3 Numerical results and analyses

With the formulae presented in previous section, we are ready to perform our numerical analysis. When we study the effects due to the MSSM with and without R-parity, we consider only one new coupling at one time, neglecting the interferences between different new couplings, but keeping their interferences with the SM amplitude. The input parameters are collected in the Appendix. The experimental upper limits at 90% confidence level (CL) [29–31] are listed in the second column of Tab. 1. and the SM predictions excluding (including) LD contributions are also listed in the third (last) column of Tab. 1. The input parameters varied randomly within 1σ variance will be used in this work.

3.1 RPV MSSM effects

First, we will consider the RPV effects and further constrain the relevant RPV couplings from relevant experimental data. As given in Sec. 2, there is only one RPV coupling product $\tilde{\lambda}'_{12k} \tilde{\lambda}'_{11k}^*$

¹S1 denotes the RPV predictions constrained from $K^+ \rightarrow \pi^+ \nu \bar{\nu}$ decay and $D^0 - \bar{D}^0$ mixing at 99.7% CL, S2 and S3 denote the RPC predictions constrained from $D^0 - \bar{D}^0$ mixing at 90% and 99.7% CL, respectively.

² $\mathcal{B}(D_d^+ \rightarrow \pi^+ \mu^+ \mu^-)_l$ denotes the branching ratios in $s \in [0.250, 0.525]$ GeV² bin, and $\mathcal{B}(D_d^+ \rightarrow \pi^+ \mu^+ \mu^-)_h$ denotes the branching ratios in $s \in [1.250, 2.000]$ GeV² bin.

Table 1: The experimental upper limits [29–33] and the theoretical predictions with 1σ error ranges of the input parameters for relevant branching ratios (in units of 10^{-10}).

Modes	Best exp. limits	SD contributions				LD contributions SM
		SM	S1 ¹	S2 ¹	S3 ¹	
$\mathcal{B}(D_u^0 \rightarrow e^+e^-)$	< 790	$\approx 10^{-13}$	$\leq 2.65 \times 10^{-7}$	[0.0027, 0.008]
$\mathcal{B}(D_d^+ \rightarrow \pi^+e^+e^-)$	< 11000	[4.84, 8.46]	[4.03, 9.31]	[1.10, 19.68]	[1.02, 21.06]	[920, 1200]
$\mathcal{B}(D_s^+ \rightarrow K^+e^+e^-)$	< 37000	[1.77, 3.08]	[1.38, 3.28]	[1.00, 7.42]	[1.00, 7.66]	[2090, 2220]
$\mathcal{B}(D_u^0 \rightarrow \mu^+\mu^-)$	< 62	$10^{-9} \sim 10^{-8}$	≤ 0.011	[0.0027, 0.008]
$\mathcal{B}(D_d^+ \rightarrow \pi^+\mu^+\mu^-)$	< 730	[4.59, 8.04]	[3.80, 8.80]	[1.06, 18.76]	[1.03, 20.10]	[920, 1200]
$\mathcal{B}(D_d^+ \rightarrow \pi^+\mu^+\mu^-)_l^2$	< 200	[0.72, 1.27]	[0.60, 1.40]	[0.10, 3.00]	[0.10, 3.22]	[0.72, 1.27]
$\mathcal{B}(D_d^+ \rightarrow \pi^+\mu^+\mu^-)_h^2$	< 260	[1.18, 2.08]	[0.99, 2.28]	[0.18, 4.81]	[0.17, 5.15]	[1.18, 2.08]
$\mathcal{B}(D_s^+ \rightarrow K^+\mu^+\mu^-)$	< 210000	[1.64, 2.84]	[1.26, 3.02]	[1.00, 6.93]	[1.00, 7.17]	[2090, 2220]

($\tilde{\lambda}'_{22k}\tilde{\lambda}'^*_{21k}$) relevant to $c \rightarrow ue^+e^-$ ($c \rightarrow u\mu^+\mu^-$) transition. Noted that $\tilde{\lambda}'_{i2k}\tilde{\lambda}'^*_{i1k}$ and $\lambda'_{i2k}\lambda'^*_{i1k}$ also contribute to $D^0 - \bar{D}^0$ mixing and $K^+ \rightarrow \pi^+\nu\bar{\nu}$ decay, respectively. In this work, we neglect the difference between λ' and $\tilde{\lambda}'$, and the bounds of relevant RPV coupling products from $D^0 - \bar{D}^0$ mixing and $K^+ \rightarrow \pi^+\nu\bar{\nu}$ decay are considered.

Relevant expressions of $D^0 - \bar{D}^0$ mixing and $K^+ \rightarrow \pi^+\nu\bar{\nu}$ decay can be found in Ref. [34] and Ref. [35–37], respectively. The latest $D^0 - \bar{D}^0$ mixing parameters $x_D = (0.56 \pm 0.19)\%$ [38] and $\mathcal{B}(K^+ \rightarrow \pi^+\nu\bar{\nu}) = (1.7 \pm 1.1) \times 10^{-10}$ [31] will be used to constrain the RPV coupling products $\tilde{\lambda}'_{i2k}\tilde{\lambda}'^*_{i1k}$. Assuming the RPV coupling products are complex, $m_{\tilde{q}} = 1000$ GeV and $m_{\tilde{g}}^2/m_{\tilde{q}}^2 \in [0.25, 4]$, we obtained $|\tilde{\lambda}'_{i2k}\tilde{\lambda}'^*_{i1k}| \in [0.0058, 0.019]$ from $D^0 - \bar{D}^0$ mixing and $|\lambda'_{i2k}\lambda'^*_{i1k}| \leq 7.95 \times 10^{-4}$ from $K^+ \rightarrow \pi^+\nu\bar{\nu}$ decay within 1.64 experimental standard deviations (at 90% CL), in other words, the RPV coupling products are excluded at 90% CL. Within three standard deviations (at 99.7% CL), we get $|\tilde{\lambda}'_{i2k}\tilde{\lambda}'^*_{i1k}| \leq 0.023$ and corresponding weak phase is free from $D^0 - \bar{D}^0$ mixing, furthermore, $|\lambda'_{i2k}\lambda'^*_{i1k}| \leq 8.97 \times 10^{-4}$ and the phase is also constrained from $K^+ \rightarrow \pi^+\nu\bar{\nu}$ decay. Considering the experimental bounds of $D^0 - \bar{D}^0$ mixing and $K^+ \rightarrow \pi^+\nu\bar{\nu}$ decay at 99.7% CL at the same time, the effective constraints from $K^+ \rightarrow \pi^+\nu\bar{\nu}$ decay at 99.7% CL, which is shown in Fig. 1 and will be used to study the RPV effects in $D_{(s)}^+ \rightarrow \pi(K)^+\ell^+\ell^-$ and $D^0 \rightarrow \ell^+\ell^-$ decays. Noted that the experimental upper limits at 90% CL listed in the

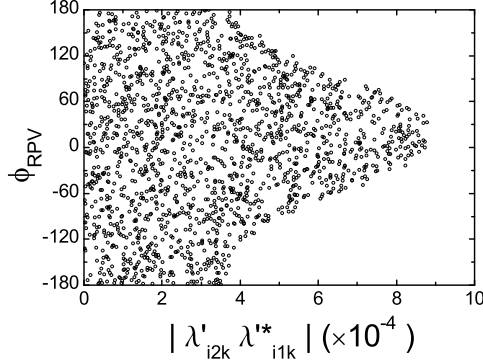


Figure 1: The allowed RPV parameter spaces from $K^+ \rightarrow \pi^+ \nu \bar{\nu}$ at 99.7% CL with 1000 GeV sfermions, and the RPV weak phase (ϕ_{RPV}) is given in degree.

second column of Tab. 1 do not give any further constraints on relevant RPV couplings.

Now we will use the constrained RPV coupling space from $K^+ \rightarrow \pi^+ \nu \bar{\nu}$ decay at 99.7% CL shown in Fig. 1 to explore the RPV coupling effects in $D_u^0 \rightarrow \ell^+ \ell^-$, $D_d^+ \rightarrow \pi^+ \ell^+ \ell^-$ and $D_s^+ \rightarrow K^+ \ell^+ \ell^-$ decays. RPV numerical results of the branching ratios are summarized in the forth column of Tab. 1. Comparing to the SM predictions excluding LD contributions, one can find that the constrained RPV couplings still have great effects on $\mathcal{B}(D_u^0 \rightarrow \ell^+ \ell^-)$, nevertheless, other four semileptonic branching ratios are not very obviously affected by the constrained RPV couplings. Comparing to the SM predictions including LD contributions in the last column of Tab. 1, the constrained RPV contributions may a little larger than the LD ones to $\mathcal{B}(D_u^0 \rightarrow \mu^+ \mu^-)$. Comparing to the experimental upper limits given in the second column of Tab. 1, one can find that all experimental upper limits are much larger than the theoretical prediction of branching ratios with SD contributions, so all upper limits listed in the second column of Tab. 1 do not give any further constraint to relevant RPV coupling products.

The sensitivities of the branching ratios to the moduli of the constrained RPV coupling products are shown in Fig. 2. Fig. 2 show us that $\mathcal{B}(D_u^0 \rightarrow \ell^+ \ell^-)$ are very sensitive to the moduli of squark exchange RPV couplings, and they are great increasing with $|\tilde{\lambda}'_{12k} \tilde{\lambda}'^*_{11k}|$ or $|\tilde{\lambda}'_{22k} \tilde{\lambda}'^*_{21k}|$. Other four branching ratios of semileptonic decays are not very sensitive to the moduli of squark exchange RPV couplings. In addition, all branching ratios are not sensitive to the relevant RPV weak phases.

Our main purpose is studying the direct CP violations of semileptonic D decays. Many

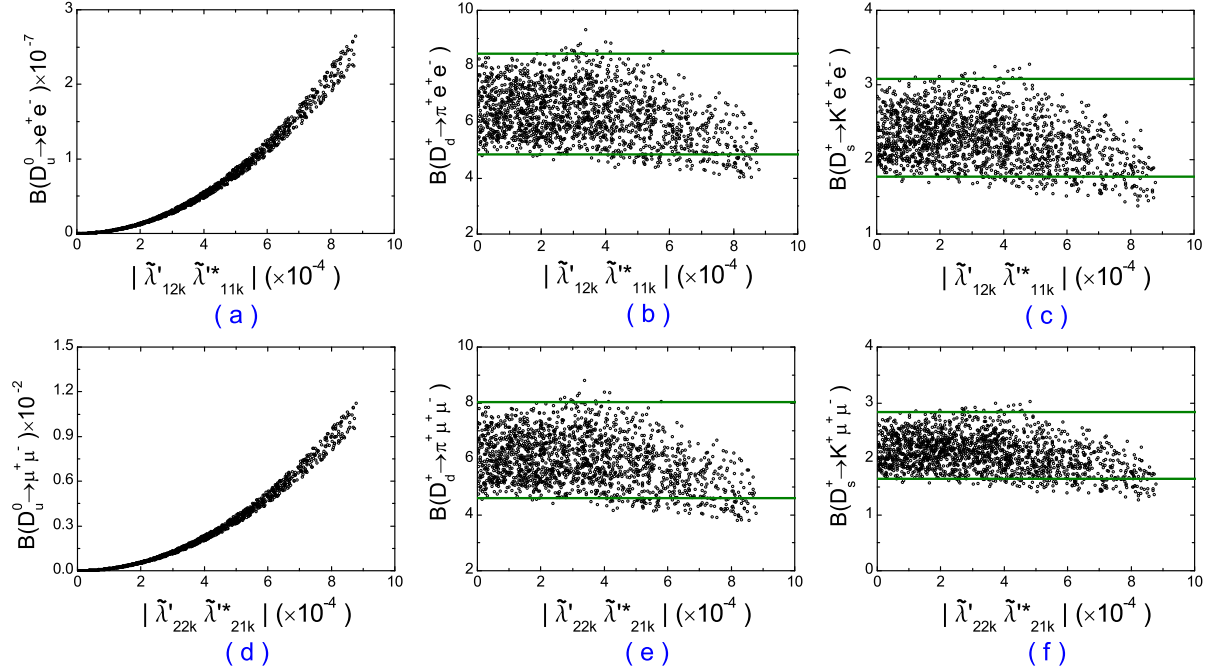


Figure 2: The RPV coupling effects due to the squark exchange on the branching ratios (in units of 10^{-10}), and the green horizontal solid lines represent the SM ranges with 1 error of the input parameters (the same in Fig. 3, Fig. 4 and Fig. ??).

theoretical uncertainties such as involved in the form factors, decay constants and CKM matrix elements are cancelled in the ratios. The RPV predictions of the direct CP violations are very dependent on the RPV coupling products, so we will not give the numerical results and only show their sensitivities to the constrained RPV coupling products. The correlations between relevant direct CP violations and the constrained RPV coupling products are shown in Fig. 3 and Fig. 4 for $D_{(s)}^+ \rightarrow \pi(K)^+ e^+ e^-$ and $D_{(s)}^+ \rightarrow \pi(K)^+ \mu^+ \mu^-$, respectively, the direct CP violation excluding (including) the LD contributions are denoted by A_{CP} (A'_{CP}), and the SM ranges of relevant direct CP violations are displayed by green horizontal solid lines. In the SM, the direct CP violations of these four semileptonic D decays are tiny, and they are at about 10^{-4} (10^{-6}) order if LD contributions excluded (included). As shown in Fig. 3 and Fig. 4, the constrained RPV couplings still have huge effects on the direct CP violations in the case of both excluded and included LD contributions, all direct CP violations could be enhanced about two orders of magnitude, and they are very sensitive to both moduli and weak phases of the RPV coupling products. In addition, the interference of resonant part of the LD contribution and

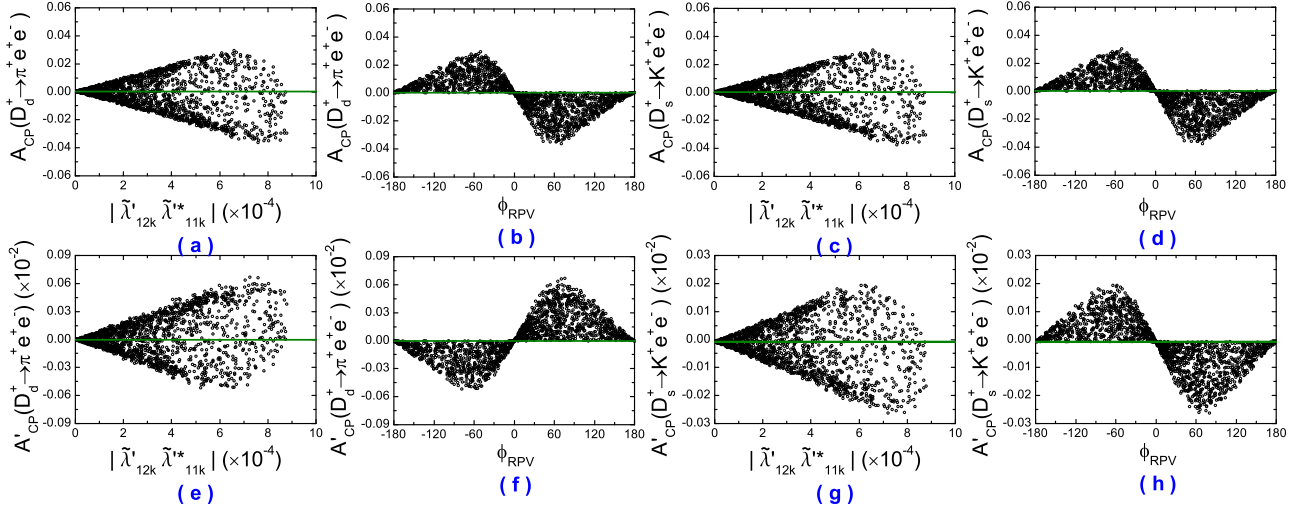


Figure 3: The effects of RPV coupling $\tilde{\lambda}'_{12k}\tilde{\lambda}^*_{11k}$ due to the squark exchange on the direct CP violations of $D_d^+ \rightarrow \pi^+ e^+ e^-$ and $D_s^+ \rightarrow K^+ e^+ e^-$ decays.

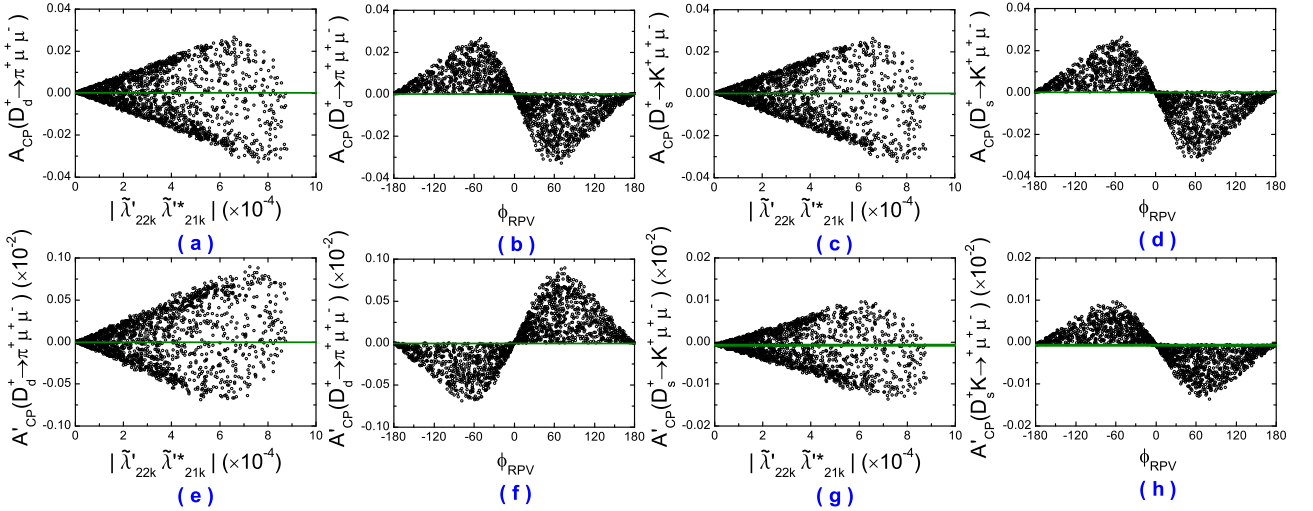


Figure 4: The effects of RPV coupling $\tilde{\lambda}'_{22k}\tilde{\lambda}^*_{21k}$ due to the squark exchange on the direct CP violations of $D_d^+ \rightarrow \pi^+ \mu^+ \mu^-$ and $D_s^+ \rightarrow K^+ \mu^+ \mu^-$ decays.

the NP affected SD contribution could reduce the direct CP violations more than one order of magnitude.

Furthermore, the constrained RPV coupling effects on the differential direct CP violations of $D_{(s)}^+ \rightarrow \pi(K)^+ \ell^+ \ell^-$ decays are also displayed in Fig. 5, and the SM predictions are given for comparison. The differential direct CP violations excluding (including) the LD contributions are

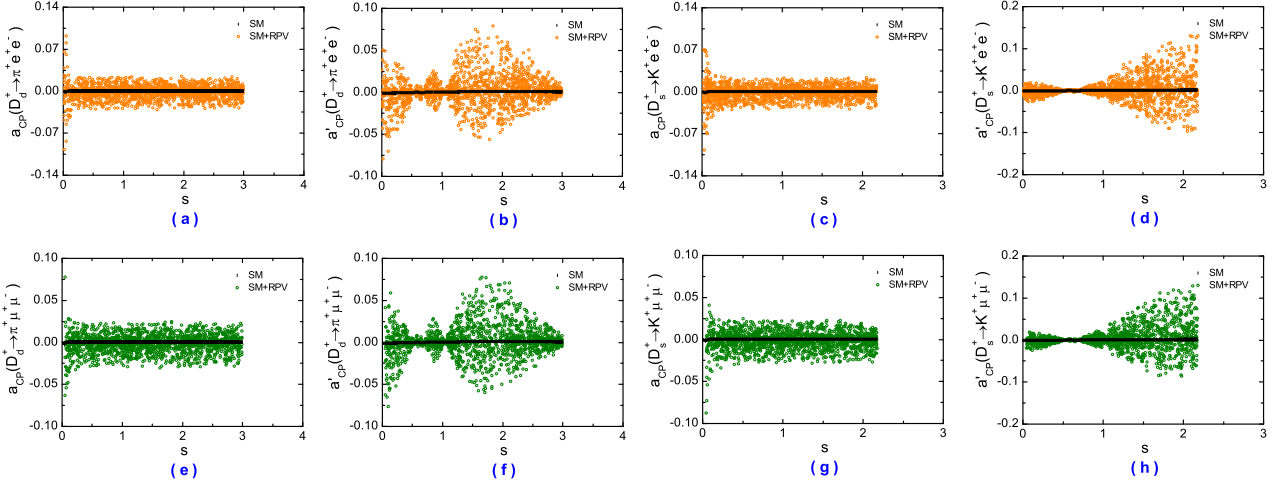


Figure 5: The RPV coupling effects due to the squark exchange in the differential direct CP violations of $D_d^+ \rightarrow \pi^+ \ell^+ \ell^-$ and $D_s^+ \rightarrow K^+ \ell^+ \ell^-$ decays.

denoted by a_{CP} (a'_{CP}). Fig. 5 shows us that the RPV couplings could have huge contributions to $a_{CP}(D_{(s)}^+ \rightarrow \pi(K)^+ \ell^+ \ell^-)$ at almost all s regions, and at middle s regions, the interference of resonant part of the LD contribution and the NP affected SD contribution could obviously reduce $a'_{CP}(D_{(s)}^+ \rightarrow \pi(K)^+ \ell^+ \ell^-)$. At all s regions, we get that $a_{CP}^{SM}(D_d^+ \rightarrow \pi^+ e^+ e^-) \in [-1.7 \times 10^{-3}, 4.67 \times 10^{-4}]$ and $a_{CP}^{SM}(D_s^+ \rightarrow K^+ e^+ e^-) \in [-1.6 \times 10^{-3}, 4.76 \times 10^{-4}]$. The constrained RPV couplings could enhance $a_{CP}^{(\prime)}$ about two orders of magnitude from their SM predictions. Noted that, for the strong phase δ_ϕ on the ϕ resonant peak appeared in the LD amplitude, $\delta_\phi = \frac{\pi}{2}, \pi$ cases also have been studied in Ref. [21], a'_{CP} for $\delta_\phi = \frac{\pi}{2}$ maybe larger a little than one for $\delta_\phi = 0, \frac{\pi}{2}$. Nevertheless, the enhancements of the RPV contributions are still totally over the LD contributions in three δ_ϕ cases.

3.2 RPC MSSM effects

Now we turn to the gluino-mediated MSSM effects in $D_{(s)}^+ \rightarrow \pi(K)^+ \ell^+ \ell^-$ decays in the framework of the MI approximation. The effects of the constrained LL insertion on $D_{(s)}^+ \rightarrow \pi(K)^+ \ell^+ \ell^-$ decays are almost negligible because of lacking the gluino mass enhancement in the decay, and they will not provide any significant effect on the branching ratios and the direct CP violations of $D_{(s)}^+ \rightarrow \pi(K)^+ \ell^+ \ell^-$ decays. The LR and RL MIs only generate magnetic operators $\mathcal{O}_{7\gamma}$ and $\mathcal{O}'_{7\gamma}$, respectively. Since LR and RL insertion contributions are enhanced by $m_{\tilde{g}}/m_b$

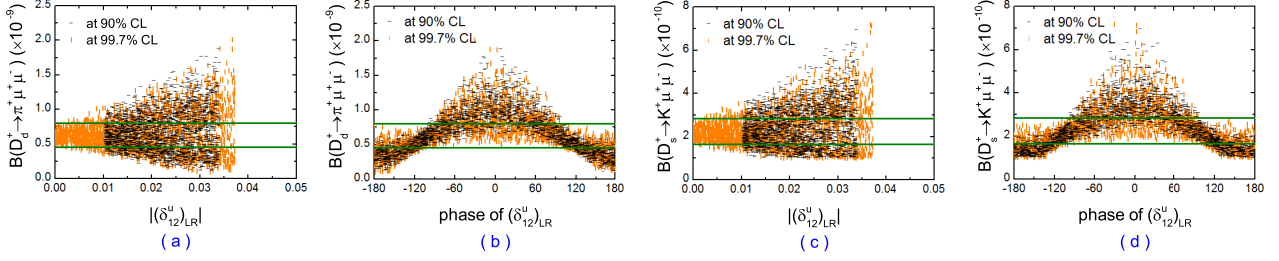


Figure 6: The LR MI effects on the branching ratios of $D_d^+ \rightarrow \pi^+ \mu^+ \mu^-$ and $D_s^+ \rightarrow K^+ \mu^+ \mu^-$ decays.

due to the chirality flip from the gluino in the loop compared with the contribution including the SM one, even a small $(\delta_{12}^u)_{LR}$ or $(\delta_{12}^u)_{RL}$ may have large effects in the decay. So we will only consider the LR and RL MI effects in this work.

Since the most stringent bounds come from $D^0 - \bar{D}^0$ mixing, we will take into account the constraints set by the $D^0 - \bar{D}^0$ mixing to investigate NP contributions in $D_{(s)}^+ \rightarrow \pi(K)^+ \ell^+ \ell^-$ decays. The latest $D^0 - \bar{D}^0$ mixing parameters, $x_D = (0.56 \pm 0.19)\%$ [38], will be used to constrain the LR and RL MIs. Using the formulae in Ref. [34], we get $|(\delta_{12}^u)_{LR,RL}| \in [0.010, 0.034]$ at 90% CL and $|(\delta_{12}^u)_{LR,RL}| \leq 0.037$ at 99.7% CL with $m_{\tilde{q}} = 1000$ GeV and $m_{\tilde{g}}^2/m_{\tilde{q}}^2 \in [0.25, 4]$. Nevertheless, the phases of $(\delta_{12}^u)_{LR,RL}$ are not restricted.

Because the LR and RL MI effects in $D_{(s)}^+ \rightarrow \pi(K)^+ e^+ e^-$ and $D_{(s)}^+ \rightarrow \pi(K)^+ \mu^+ \mu^-$ are same as each other, here we take the LR insertion effects in $D_{(s)}^+ \rightarrow \pi(K)^+ \mu^+ \mu^-$ as an example. RPC numerical results of the branching ratios constrained from $D^0 - \bar{D}^0$ mixing at 90% and 99.7% CL are summarized in the fifth and sixth columns of Tab. 1, respectively. The constrained RPC MI couplings still have obvious effects on four semileptonic branching ratios, but their contributions are much smaller than the LD ones.

The LR MI effects on $\mathcal{B}(D_{(s)}^+ \rightarrow \pi(K)^+ \mu^+ \mu^-)$ and $A_{CP}(D_{(s)}^+ \rightarrow \pi(K)^+ \mu^+ \mu^-)$ are shown in Fig. 6 and Fig. 7, respectively. As shown in Fig. 7, $A_{CP}(D_{(s)}^+ \rightarrow \pi(K)^+ \mu^+ \mu^-)$ are obviously affected by the constrained LR MI constrained from $D^0 - \bar{D}^0$ mixing, their RPC predictions could be much larger than their SM ones. In addition, the branching ratios and the direct CP violations are quite sensitive to both modulus and weak phase of $(\delta_{12}^u)_{LR}$.

The constrained LR MI effects on the differential branching ratios and the differential direct CP violations of $D_d^+ \rightarrow \pi^+ \mu^+ \mu^-$ and $D_s^+ \rightarrow K^+ \mu^+ \mu^-$ decays are displayed in Fig. 8. As shown

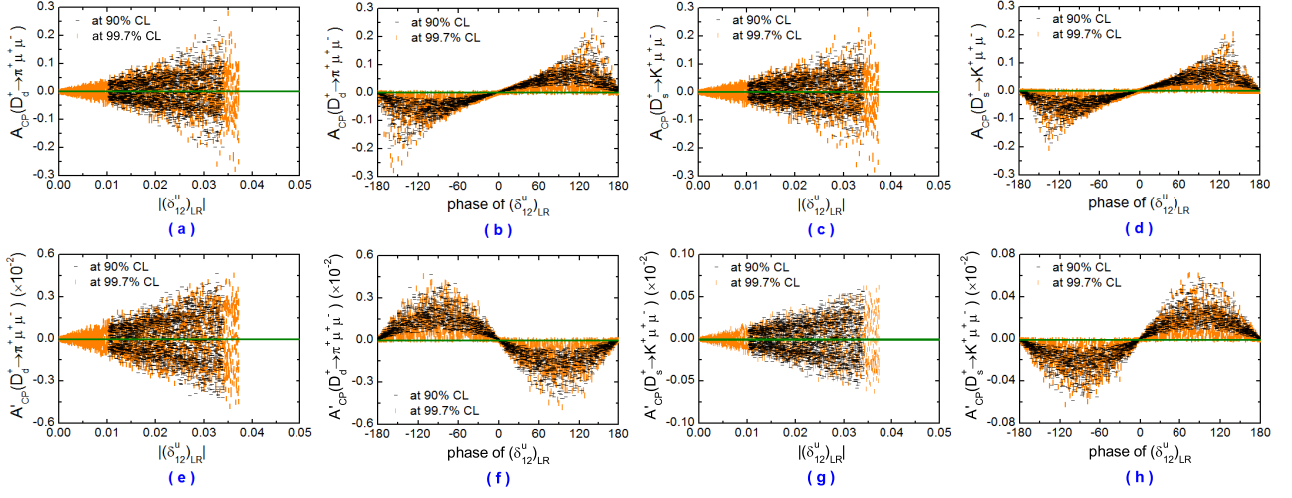


Figure 7: The LR MI effects on the direct CP violations of $D_d^+ \rightarrow \pi^+ \mu^+ \mu^-$ and $D_s^+ \rightarrow K^+ \mu^+ \mu^-$ decays.

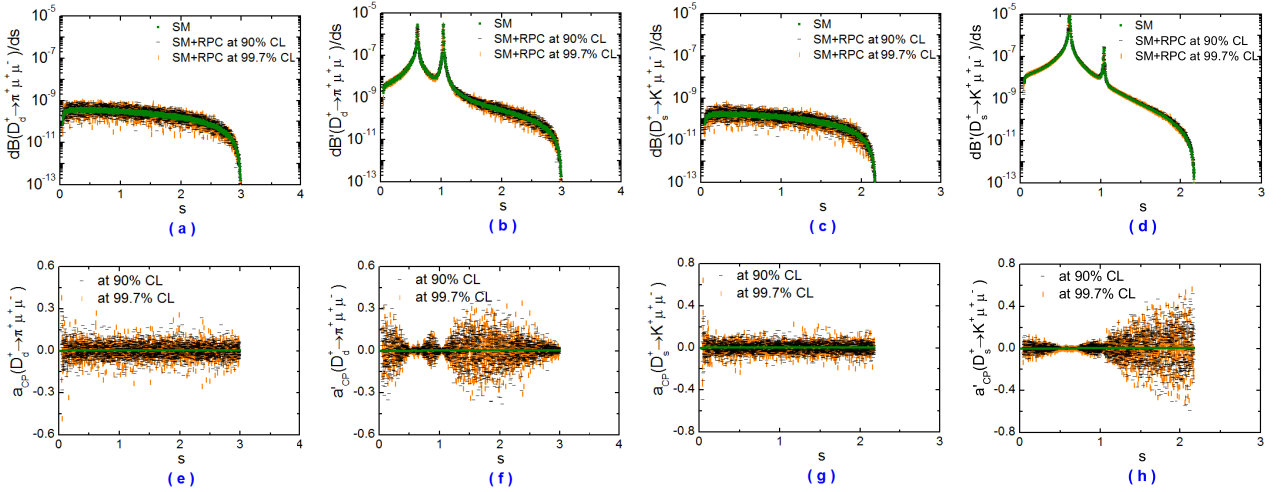


Figure 8: The LR MI effects on the differential branching ratios and the differential direct CP violations of $D_d^+ \rightarrow \pi^+ \mu^+ \mu^-$ and $D_s^+ \rightarrow K^+ \mu^+ \mu^-$ decays.

in Fig. 8 (a-d), the differential branching ratios could be slightly affected by the constrained LR MI. From Fig. 8 (e) and (g), one can see that the differential direct CP violations could be significantly affected at all s regions by the constrained LR MI, nevertheless, as displayed at middle s region of Fig. 8 (f) and (h), the interference of resonant part of the LD contribution and the RPC affected SD contribution could hugely reduce the differential direct CP violations more than two orders of magnitude.

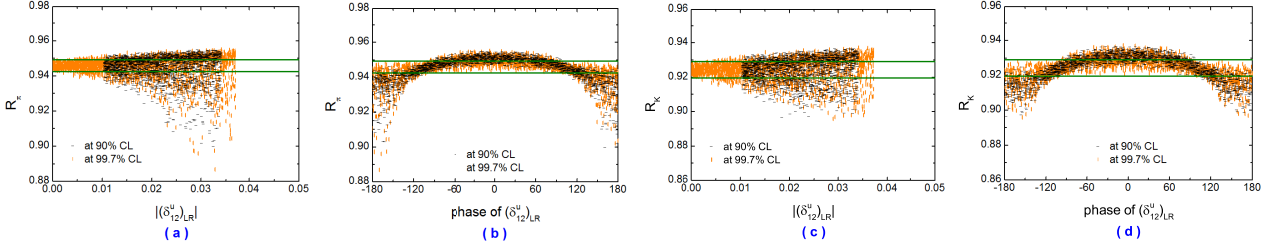


Figure 9: The LR MI effects on the ratios of $\mathcal{B}(D_{(s)}^+ \rightarrow \pi(K)^+ \mu^+ \mu^-)$ and $\mathcal{B}(D_{(s)}^+ \rightarrow \pi(K)^+ e^+ e^-)$ decays.

Moreover, the ratios of decay branching ratios of $D_{(s)}^+ \rightarrow \pi(K)^+ \ell^+ \ell^-$ into dimuons over dielectrons are also studied since the branching ratios of $D_{(s)}^+ \rightarrow \pi(K)^+ \ell^+ \ell^-$ are affected by the same LR and RL MI parameters. The constrained LR MI effects on R_π and R_K are displayed in Fig. 9. One can see that the constrained LR MI still has great effects on these ratios, which are very sensitive to both modulus and weak phase of $(\delta_{12}^u)_{LR}$. When the modulus and absolute phase of $(\delta_{12}^u)_{LR}$ are large, R_π and R_K may have small values.

4 Summary

In this work we have performed a brief study of the RPV coupling effects and the RPC MI effects in the MSSM from $D_{(s)}^+ \rightarrow \pi(K)^+ e^+ e^-$, $\pi(K)^+ \mu^+ \mu^-$ and $D^0 \rightarrow e^+ e^-$, $\mu^+ \mu^-$ decays. Considering the theoretical uncertainties and using the strong constraints on relevant supersymmetric parameters from $D^0 - \bar{D}^0$ mixing or $K^+ \rightarrow \pi^+ \nu \bar{\nu}$ decay, we have investigated the sensitivities of the dileptonic invariant mass spectra, branching ratios, differential direct CP violation, direct CP violation and ratios of $D_{(s)}^+ \rightarrow \pi(K)^+ \mu^+ \mu^-$ and $D_{(s)}^+ \rightarrow \pi(K)^+ e^+ e^-$ decay rates to the survived RPV and RPC coupling spaces.

We have found that, after satisfying the current experimental bounds of $D^0 - \bar{D}^0$ mixing and $K^+ \rightarrow \pi^+ \nu \bar{\nu}$ decay at 99.7% CL, left-handed squark exchange RPV couplings have significant effects on $\mathcal{B}(D^0 \rightarrow \ell^+ \ell^-)$, differential direct CP violation and direct CP violation of $D_{(s)}^+ \rightarrow \pi(K)^+ \ell^+ \ell^-$ semileptonic decays. The direct CP violations of $D_{(s)}^+ \rightarrow \pi(K)^+ \ell^+ \ell^-$ are sensitive to both moduli and phases of relevant RPV coupling products.

As for the RPC MI effects, we found that the constrained LR and RL insertions from $D^0 - \bar{D}^0$ mixing at 90% CL and 99.7% CL could significantly affect the branching ratios and

the direct CP violations of $D_{(s)}^+ \rightarrow \pi(K)^+ \ell^+ \ell^-$ as well as ratios of $D_{(s)}^+ \rightarrow \pi(K)^+ \mu^+ \mu^-$ and $D_{(s)}^+ \rightarrow \pi(K)^+ e^+ e^-$ decay rates, and they are very sensitive to both moduli and phases of relevant LR and RL mass insertion couplings.

With the running of LHC experiment, the prospects of measuring $D_{(s)}^+ \rightarrow \pi(K)^+ \ell^+ \ell^-$ and $D^0 \rightarrow \ell^+ \ell^-$ decays could be realistic. We expect that future experiments will significantly strengthen the allowed parameter spaces for RPV couplings and RPC MIs. Our predictions on related the direct CP violations of semileptonic D decays and the ratios of $D_{(s)}^+ \rightarrow \pi(K)^+ \mu^+ \mu^-$ and $D_{(s)}^+ \rightarrow \pi(K)^+ e^+ e^-$ decay rates could be very useful for probing supersymmetric effects in future experiments.

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Appendix: Input parameters

The input parameters are collected in Table 2. We have several remarks on the input parameters:

- Wilson coefficients: The SM Wilson coefficients C_i^{SM} are obtained from the expressions in Ref. [5].
- CKM matrix element: For the SM predictions, we use the CKM matrix elements from the Wolfenstein parameters of the latest analysis within the SM in Ref. [39], and for the SUSY predictions, we take the CKM matrix elements in terms of the Wolfenstein parameters of the NP generalized analysis results in Ref. [39].
- Form factors: We use the results of form factors in Ref. [40]. In our numerical data analysis, the 10% uncertainties induced by $F(0)$ are also considered.

Table 2: Values of the theoretical input parameters. To be conservative, we use all theoretical input parameters at 68% CL in our numerical results.

$m_W = 80.385 \pm 0.015$ GeV, $m_{D_u} = 1.865$ GeV, $m_{D_d} = 1.870$ GeV, $m_{D_s} = 1.969$ GeV,	
$m_{D_d^*} = 0.2010$ GeV, $m_{D_s^*} = 2.112$ GeV, $m_{\pi^+} = 0.140$ GeV, $m_{K^+} = 0.494$ GeV, $m_{\rho^0} = 0.775$ GeV,	
$m_\omega = 0.783$ GeV, $m_\phi = 1.019$ GeV, $m_t = 173.5 \pm 1.0$ GeV, $m_b = 4.78 \pm 0.06$ GeV,	
$m_c = 1.67 \pm 0.07$ GeV, $\overline{m}_b(\overline{m}_b) = (4.18 \pm 0.03)$ GeV, $\overline{m}_s(2\text{GeV}) = (0.095 \pm 0.005)$ GeV,	
$\overline{m}_d(2\text{GeV}) = (0.0048 \pm 0.0003)$ GeV, $m_e = 5.11 \times 10^{-4}$ GeV, $m_\mu = 0.106$ GeV;	
$\tau_{D_u} = (0.4101 \pm 0.0015)$ ps, $\tau_{D_d} = (1.040 \pm 0.007)$ ps, $\tau_{D_s} = (0.500 \pm 0.007)$ ps,	
$\Gamma_{\rho^0} = (0.1462 \pm 0.0007)$ GeV, $\Gamma_\omega = (8.49 \pm 0.08) \times 10^{-3}$ GeV, $\Gamma_\phi = (4.26 \pm 0.04) \times 10^{-3}$ GeV;	
$f_{D_u} = 0.2067 \pm 0.0089$ GeV, $f_{K^+} = (0.15610 \pm 0.00083)$ GeV, $f_\pi = (0.13041 \pm 0.00020)$ GeV.	[31]

The Wolfenstein parameters for the SM predictions:

$A = 0.827 \pm 0.013$, $\lambda = 0.22535 \pm 0.00065$, $\bar{\rho} = 0.132 \pm 0.021$, $\bar{\eta} = 0.350 \pm 0.014$.

The Wolfenstein parameters for the SUSY predictions:

$A = 0.802 \pm 0.020$, $\lambda = 0.22535 \pm 0.00065$, $\bar{\rho} = 0.147 \pm 0.048$, $\bar{\eta} = 0.370 \pm 0.057$.

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